Non-rigid Surface Registration using Cover Tree based Clustering and Nearest Neighbor Search

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Abstract: We propose a novel non-rigid registration method that computes the correspondences of two deformable surfaces using the cover tree. The aim is to find the correct correspondences without landmark selection and to reduce the computational complexity. The source surface S is initially aligned to the target surface T to generate a cover tree from the densely distributed surface points. The cover tree is constructed by taking into account the positions and normal vectors of the points and used for hierarchical clustering and nearest neighbor search. The cover tree based clustering divides the two surfaces into several clusters based on the geometric features, and each cluster on the source surface is transformed to its corresponding cluster on the target. The nearest neighbor search from the cover tree reduces the search space for correspondence computation, and the source surface is deformed to the target by optimizing the point pairs. The correct correspondence of a given source point is determined by choosing one target point with the best correspondence measure from the k nearest neighbors. The proposed energy function with Jacobian penalty allows deforming the surface accurately and with less deformation folding.

1 INTRODUCTION

Iterative closest point algorithm (ICP) has been widely used for registration of the surfaces (Besl and McKay 1992; Zhang 1992). The ICP efficiently calculates the transformation between two surfaces by minimizing the Euclidean distance of the correspondent point pairs. However, this distance based measure can lead the optimization to the local minima when the two surfaces are not close enough. Another limitation of ICP algorithm is that it requires searching all the points of a surface to determine the best correspondence for a point of another surface. Therefore, its time complexity is $O(n^2)$.

The correspondence computation can be accelerated using efficient nearest neighbor (NN) search algorithms (Greenspan and Godin 2001). The k-d tree has been widely used to limit the search space to one set which is the nearest (Bentley 1975). The k-d tree is a binary tree that is built by repeatedly dividing the space into subspaces using hyper planes. The k-d tree construction is simple and it is quite efficient especially for low dimensional data. However, its axis-aligned point division regardless of point distribution can result in poor search performance (Kumar et al. 2008). Greenspan and Godins proposed a variant of k-d tree with spherical triangular constraint that specifies the neighborhood which lie within a sphere of radius ε (Greenspan and Godin 2001). Kumar et al. used the vantage point tree (vp-tree) that divides the space by using hyper shells with increasing radius instead of using hyper planes (Kumar et al. 2008).

It is important to find the correct correspondence as well as to accelerate the computation. To determine the robust correspondences, Anguelov et al. proposed a joint probabilistic model that enforces the correlation between all correspondences in terms of geodesic distance and penalizes the stretching and twisting of the links between points (Anguelov et al. 2005). Huang et al. also constrained the geodesic distances between points to be preserved by the correspondences (Huang et al. 2008). They initialized the candidate correspondences of all points by finding the closest in Euclidean space and

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feature space and pruned inconsistent mappings based on the geodesic distance constraint.

In this paper, we propose an accurate non-rigid ICP registration method that finds the correct correspondences and reduces the computation complexity with an efficient tree search. We address the two challenges of the naïve ICP algorithm: the optimization to the local minima and high time complexity for correspondence computation. The main idea of our method is to reduce the number of possible correspondences from two surfaces by using a hierarchical cover tree structure and find the point pairs with the best correspondence measures. A cover tree is constructed from the points of the two surfaces which are target T and initially matched source S. Given a point *p* on S, the candidate corresponding points on T are determined by traversing the cover tree and finding the nearest neighbors from the tree instead of searching all the points on T. This correspondence search is applied for rigid ICP as well as non-rigid ICP. For rigid registration, the nearest neighbors are determined by dividing the tree nodes into several clusters. The search space is limited to the leaf nodes of the cluster which *p* belongs to. For non-rigid ICP, k-NN search is performed on the tree to find the k-nearest leaf nodes from T. We propose a correspondence measure which takes into account local geometric similarity.

For registration of the two surfaces, energy minimization frameworks that minimize the distance function between the surfaces have been proposed. The distance energy function has been used to fit one surface to another in conjunction with marker error term between manually selected feature correspondences. Allen et al. proposed a non-rigid ICP algorithm that determines local affine transformation per point by optimizing the distance function (Allen et al. 2003). They added stiffness term to force neighboring points to have similar transformations and marker error term to avoid the optimization to the local minima. Amberg et al. optimized a similar energy function for fixed stiffness and correspondences (Amberg et al. 2007). They demonstrated accurate registration results for the surface with a large missing region. Pauly et al. optimized a distance function that calculated the sum of the distances between a surface point and local neighborhood of a point on another surface (Pauly et al. 2005). Li et al. optimized the correspondence as well as the deformation parameters (Li et al. 2008). They also optimized a confidence weight of each node in order to determine the correspondences reliably and deal with the partial overlap problem of the surfaces. In these related works, the stiffness term has been effectively used to regularize the deformation by minimizing the difference between the deformation vectors of the adjacent points on the surface. However, it does not deal with the problem of the deformation folding which has the negative Jacobian determinant of the deformation and results in crossing of the adjacent deformation vectors.

To register the two surfaces with less deformation folding while minimizing the distance and stiffness, we propose a new energy function that consists of the terms: fitting, stiffness, and Jacobian penalty. The fitting term finds the deformation vectors that minimize the error distances between two corresponding point sets and the stiffness term regularizes the deformation by minimizing the difference between the deformation vectors of the adjacent points on the surface. The Jacobian penalty term penalizes negative Jacobian determinant of the deformation (Rueckert et al. 2006). The Jacobian matrix of the transformation has been applied to guarantee the invertibility of the transformation mainly for image registration (Vercauteren et al. 2009; Rohlfing et al. 2001). We adapt this penalty to prevent the deformation folding on the surface.

The non-linear optimization of the two point sets in the non-rigid registration of the surfaces is computationally expensive since the number of points on a surface is usually several thousands. For efficient optimization, previous works proposed reduced deformable models which divided the surface into many small patches and transformed them rigidly. (Huang et al. 2008; Li et al. 2008; Chang and Zwicker 2009; Liu et al. 2009).

To closely match the two surfaces and thus accelerate the optimization of the non-rigid ICP, we propose a cluster-based locally rigid registration that splits the two surfaces into clusters and transforms each cluster on S to the corresponding cluster on T by applying rigid ICP. All the points on T that belong to the same cluster with the given source point *p* considered in the corresponding cluster on T. We refer to this registration method as a cluster-based ICP. The previous reduced deformable models have used regularly sampled points (Li et al. 2008; Sumner et al. 2007), voxel grid structure (Chang and Zwicker 2009), or clusters (Huang et al. 2008; Liu et al. 2009). In the aspects of using the hierarchical clustering for registration, our cluster-based ICP is similar to that of (Huang et al. 2008; Liu et al. 2009). However, our hierarchical clustering is based on cover tree and our cluster-based ICP is applied to initially match two input surfaces not for deformable registration.

2 METHOD

The non-rigid registration of the surfaces aims to find the correct correspondences between S and T and align S to T accurately by using the correspondences. We propose a new non-rigid registration method of the surface to achieve these two goals. The proposed method consists of four steps which are initial alignment, construction of the cover tree, cluster-based ICP, and non-rigid ICP registration. The two input surfaces are initially matched by aligning the points with minimum z depth, which are the positions of the nose tip in our tested dataset, and scaling the surface S according to the maximum ranges of the points on S and T. After initial alignment, we construct a cover tree from the points of both surfaces and use it for hierarchical clustering and k-NN in the correspondence computation of the cluster-based ICP and non-rigid ICP, respectively. For the cluster-based ICP, we first find the correspondence of each point on S among the points in the same cluster which comes from T and has the best correspondence measure. Once the corresponding point sets on the two surfaces have been determined, each cluster on S is locally transformed to T by minimizing the error between the two point sets. In the non-rigid ICP registration, the candidate correspondences of a given point on S are computed by looking for its k-NN in the cover tree, which originate from T. A correct correspondence is chosen by finding the best correspondence measure among the k nearest points. With the two correspondent point sets, the proposed method deforms S to T by optimizing the energy function that includes a fitting term, a stiffness term, and a Jacobian penalty term.

2.1 Use of Cover Tree

The cover tree is a leveled tree where levels are decreased as the tree is descended (Beygelzimer et al. 2006). Each node in the tree corresponds to a point in dataset P. Let P_i denote the points of P at level i. The cover tree has three properties of nesting, covering, and separation. The nesting property indicates that a point at a level i should appear at all the levels beneath it. The covering property satisfies the condition that the distance between a point q in P_{i-1} and its parent in P_i is at most 2^i. The separation property meets the condition that the distance between two distinct points at the same level is at least 2^i (Fig. 2).

The construction of the cover tree takes $O(n)$ space and $O(c^6\ n \log n)$ time complexity. The time complexity does not only depend on the number of points of the dataset n, but also on the expansion constant c. Expansion constant is defined as the ratio of the points in a sphere with the maximum radius r over the points in a sphere with the radius of $r/2$ (Beygelzimer et al. 2006).

In this paper, we adopt the cover tree data structure for hierarchical clustering and k-NN search. By using the cover tree with its nesting, covering, and separation properties, the problem of finding the correct correspondence in the ICP registration is reduced from searching all the points on T to searching a subset of the points. This subset of the points is represented as a cluster of the points within Ɛ-radius for cluster-based ICP and as k nearest points for non-rigid ICP.

2.1.1 Cover Tree Construction using Distance and Surface Normal

Originally, the cover tree is constructed by taking into account the distance between the points. Even though the distance based cover tree can be used for clustering points, it is difficult to obtain meaningful clusters from surfaces that are not flat and have complex geometric shapes. In order to subdivide the surface into meaningful clusters such that the points in each cluster have similar geometric features, we extend the tree construction method by considering the angular difference between the surface normal vectors of the points as well as the Euclidean distance between the points. We define our new distance metric as a function f of two terms; one for the Euclidian distance between two points x and y, d, and the other for the angle θ between the normal vectors, as described in $f(x,y)=d(x,y)+\lambda(1-\cos\theta(x,y))$. As the angle θ increases, the value of the second term increases which makes the value of f increase. As a result, the two points are located far away from each other in the cover tree. If the two points have the same surface normal, only d affects the value of f and the two points will be located in the cover tree according to the distance. The parameter λ is a weighting factor that controls the effect of the angle θ. The value λ is determined according to the features of the surface. As λ increases, the effect of the second term becomes larger and the surface will be clustered into points with similar geometric features. However, setting λ to max can partition the points that belong to one anatomical feature into many clusters with respect to the normal variations in that feature as shown in Fig. 3. We set λ to 0.05 experimentally to divide the surface into meaningful clusters which correspond to anatomical features. By using the function *f* in the tree construction, a parent *q* for a new point *p* should satisfy the following condition $f(p,q) \leq 2^i$. Here, *i* is the level of the cover tree where *q* is located. The span of the cover tree, including the number of levels, is affected by *f*. This proposed function *f* satisfies the properties of a distance function in a metric space.

2.1.2 The Use of the Cover Tree for Hierarchical Clustering

After the cover tree is built from the points of the two surfaces, the points are divided into k disjoint clusters by cutting the tree at the level *i* with k nodes such that each of the k nodes is a root of a sub-tree and each sub-tree is considered a cluster as shown in Fig. 2. As a result, each cluster denoted by C_j is rooted at its center, and the neighbor points within a radius $\varepsilon = 2^i$ from the center correspond to the leaf nodes of the sub-tree.

2.1.3 The Use of the Cover Tree for NN Search

The correspondence computation can be formulated as a NN search problem in naïve ICP due to the fact that it is based on the distance between the points. In the NN search problem, the dataset *P* of *n* points is pre-processed such that one can find the nearest neighbor point *p* of a given query point *q* with the minimum distance $d(q, p)$. The constructed cover tree is used to find the k-NN points. Given a point $p \in P$, the nearest points are determined by searching the children list *Q* of *p* and finding a point with the minimum error $d(p,Q) = min_{q \in Q}\, d(p,q)$. The error is calculated with respect to the distance and angular difference of the normal vectors. The exact k-NN points are determined by sorting the errors between *p* and *q* and finding the k points with the smallest error. The NN search takes $O(c^{12} \log n)$ time (Beygelzimer et al. 2006).

2.2 Cluster-based ICP Registration

Rigid registration has been applied to compensate the translational and rotational mismatch between two surfaces. Recently, local rigid or affine registration was used for reduced deformable model (Chang and Zwicker 2009; Huang et al. 2008; Li et al. 2008). By reducing the degrees of freedom for optimization while considering the rigidity, S can be deformed to T quickly and accurately. The proposed cluster-based ICP method calculates the local rigid transformations from several clusters which are partial patches of the surfaces. It is used to provide a good initial match for non-rigid registration.

2.2.1 Correspondence Computation

To find the correspondence of a point *p* on S among the points on T, two surfaces are divided into multiple clusters by cover tree based hierarchical clustering described in Section 2.1.2. Only the points in the same cluster with the *p* are considered as candidates. For all candidate points *q* which come from T, the correspondence measure is calculated using E_{Corr} *as in* Eq. 1 and the point with the minimum measure E_{Corr} is determined as the correspondence of *p*:

$$E_{Corr}(p,q) = E_{Dist} + \alpha E_{Normal} + \beta E_{Isometric} \tag{1}$$

The first term E_{Dist} is used to find the closest point by calculating the Euclidean distance between two points. The correspondence computation only using Euclidean distance is not sufficient even though rough correspondence is established by cover tree based hierarchical clustering. To find more reliable correspondence, we calculate two local geometric measures. E_{Normal} which is the angle between the normal vectors is used to penalize the points in the opposite surface direction. $E_{Isometric}$ is defined to enforce the two corresponding points that have similar connectivity with the adjacent points. This measures the absolute difference between the length of the connecting edges of *p* and that of the connecting edges of *q*. The parameters, α and β, control the effect of E_{Normal} and $E_{Isometric}$ against E_{Dist}. As these parameters for local geometric features are larger, the effect of E_{Dist} decreases and the determined corresponding point sets can slow down the optimization. We set α and β to 0.05 experimentally in order to find the correspondence that has similar geometric features while obtaining the reasonable optimization performance.

2.2.2 Optimization

The transformation of each cluster is calculated by minimizing the rigid registration error using Eq. 2, where *R* and *Tr* are the rotation matrix and translation vector, and p_i and q_i are the points on S and T:

$$E_R = \sum_{i=1}^{n} \left\| p_i - (Rq_i + Tr) \right\|^2 \tag{2}$$

To reduce the discontinuity of the transformations between clusters, the transformation of each point is calculated by weighted averaging the rigid transformations of the k nearest clusters. The weight for each cluster is calculated in proportion to the distance between the point and the center of each cluster.

2.3 Non-rigid Registration

The proposed non-rigid ICP registration consists of two steps. First, the correspondence of a given source point is computed by searching k-NN in the cover tree and finding a point with the minimum correspondence measure. Second, once the two corresponding point sets are determined, S is deformed to T by minimizing the proposed energy term so that the deformation is both accurate and smooth, and has less deformation folding.

2.3.1 Correspondence Computation

For non-rigid registration of S to T, it is very important to determine the correspondences reliably and efficiently. To address this challenge, we propose a method for correspondence computation of non-rigid ICP registration. To find the correspondence of a given point *p* on S, the search space is limited to k-nearest points on T by using the cover tree based k-NN search as described in Section 2.1.3. Only k points which are the nearest from *p* are considered as candidates. As k is larger, more points are included as candidates and the computation time of E_{Corr} will increase. When k is too small, possible candidates that might have the best correspondence measure could be missed even though the computation will be faster. We set k to 10 experimentally to find the best correspondence among the sufficient number of candidates while reducing the computation time. For all candidate points, the correspondence measure is calculated using Eq. (1) as described in Section 2.2.1, and the point with the minimum measure E_{Corr} is determined as a correspondence.

2.3.2 Optimization

After determining two correspondent point sets from S and T as *P* and *P'*, respectively, the points in *P* are deformed to the points in *P'*. The deformation *D* is calculated by minimizing the registration error E_N described in Eq. 3:

$$E_N = \sum_{i=0}^{N} \omega_i E_{Fit} + \gamma E_{Smooth} + \delta E_{Jacobian} \quad (3)$$

The first term E_{Fit} measures the accuracy of alignment by calculating the distance between *P'* and *D(P)*. The second error term E_{Smooth} regularizes the deformation by minimizing the sum of differences of the deformation between adjacent points as shown in Eq. 4:

$$E_{Smooth}(D(p_i)) = \sum_{p_j \in N(p_i)} \left\| D(p_j) - D(p_i) \right\| \quad (4)$$

The third term $E_{Jacobian}$ regularizes the deformation by assigning penalty to the points with the negative Jacobian determinant. To impose penalty to the points with negative Jacobian and avoid the folding of the deformation, $E_{Jacobian}$ is defined by Eq. 5:

$$E_{Jacobian}(D(p_i)) = c \log(1 - Det(J(D))) \quad (5)$$

where *Det(J)* is the determinant of the Jacobian matrix *J*, and *c* is the constant that adjusts the effect of the negative Jacobian term. The constant *c* is proportional to the distance between p_i and its farthest neighbor. This Jacobian penalty term is applied only for the points with the negative Jacobian. To minimize E_N between two corresponding point sets, the Levenberg Marquardt optimization algorithm is applied (Marquardt 1963). γ and δ are the parameters that adjust the effect of stiffness term and Jacobian term, respectively. If the stiffness parameter γ is small, the optimization converges quickly to the closest point based on the fitting term. However, the surface mesh becomes very irregular and bumpy. As γ is larger, the deformation is smoother but the optimization becomes slower and the surface may shrink. We set γ to 1. The parameter δ for Jacobian term is set to 1 if the point has a negative Jacobian. Otherwise the value is set to 0. The optimization ends when the termination condition is met. If the reduced error measure after each iteration *i*, E_N^i - E_N^{i-1}, is less than 5% of the error measure E_N^i, it is considered that the optimization converges to the optimum. By penalizing the deformation with stiffness term and Jacobian term, the proposed optimization regularizes the deformation so that the deformed surface has smooth deformation with less folding.

3 EVALUATION METHODS

To evaluate the proposed method, we tested three different datasets; CT-simulated CT dataset, CT-Kinect dataset, CT-CT dataset. For simulated dataset, we extracted 3D surface from CT (Computed Tomography) scans using Marching Cube Algorithm (Lorensen and Cline 1987) and used it as the source surface S. We simulated the target surface T by warping the jaw and nose of S using thin-plate spline warping (Bookstein 1989). For CT-Kinect dataset, S was generated from the 2D color image and depth map obtained from Microsoft Kinect camera. A 3D surface with color was generated from depth map by back-projecting the 2D pixel positions. The surface T was extracted from the CT scans. For CT-CT dataset, we extracted S and T from two CT datasets which were acquired from two different subjects. We also tested the registration accuracy of noisy CT datasets in order to demonstrate the robustness of the proposed method to noisy dataset. Table 1 shows the number of points in each tested dataset.

To demonstrate the effect of the proposed cover tree-based clustering method, we compared our clustering method with two k-means clustering algorithms that initialize the cluster centers in different ways. The first uses manually selected initial centers (k-means manual) (Lloyd 1982) and the second detects the centers automatically using k-means++ algorithm (k-means++) (Arthur and Vassilvitskii 2007).

To evaluate the effect of the proposed correspondence computation using cover tree, we compared the proposed method with the naïve ICP algorithm, ICP algorithm with k-means manual, ICP algorithm with k-means++ in aspects of the registration accuracy. To compare the three clustering methods in the same condition, we used the same number of the clusters obtained from the proposed clustering method for k-means manual and k-means++ seeds number. The proposed method cut the cover tree at depth equals to 3. We implemented the naïve ICP algorithm that finds the correspondence of a point *p* on S by searching the point with the minimum distance from *p* among all the points on T. For cluster-based registration, the k-means manual and k-means++ clusters are used in comparison with proposed method. For non-rigid ICP correspondence, a cover tree based NN search was applied to the results of the cluster-based ICP of three different clustering algorithms along with naïve ICP.

We visualized the color-coded error surfaces in which the color of each surface point indicates its own error measure. The point was colored red if the depth of a point on T is closer than the depth of the corresponding point on the deformed S. The point was colored green in the opposite case.

4 RESULTS

Fig. 1 shows the result of the proposed registration method for three different datasets. The first row shows that the overall shape of S near jaw and nose was deformed to the simulated surface T accurately. The details of the surface such as lips and eyes were not preserved due to the lower resolution of original surface and the effect of stiffness term during optimization. The second row shows the noisy surface S acquired from Microsoft Kinect camera was deformed to T closely. Even though the two surfaces obtained from different subjects by different devices have distinctively different nose and mouth shapes, the color-coded error map shows that entire face of S was deformed to T correctly. There was a subtle difference between T and deformed S near teeth and nose of the subject. The third row shows that the two face surfaces acquired from two different subjects were registered accurately after applying the proposed method. The surface S with open mouth and low nose tip was deformed to the surface with closed mouth and high nose tip that has little difference with T.

The registration errors of three methods with clustering were lower than that of Naïve ICP. The proposed method with cover tree led to the lowest reduction rate against Naïve ICP. Especially in Kinect-CT dataset with irregular point distribution, the reduction rate of the proposed method was 28% while those of k-means manual and k-means++ were 26% and 22%, respectively. There was no significant difference in simulated CT and CT-CT datasets which have relatively regular point distributions. Fig. 4 shows the registration accuracy of the noisy datasets compared to original dataset. The registration errors of the k-means manual and k-means++ increased in two noisy datasets, as opposed to the cover tree method. The registration error decreased in simulated dataset with the cover tree method. Also, the increase of the error was the smallest with cover tree in CT-CT dataset.

The proposed method which optimizes Jacobian penalty term led to the smallest percentage of the negative Jacobian in simulated dataset and CT-CT datasets as shown in Table 2.

Table 3 shows the processing time for clustering the points and finding correspondences by using three different methods. The time for cover tree based clustering was shorter than those for two k-means clustering methods. Once the cover tree is constructed in a pre-processing step, the proposed clustering method only takes time for cutting the tree at a specific level and labelling the points. While, k-means manual and k-means++ require iterative calculation of the distances until stabilization.

5 CONCLUSIONS

In this study, we proposed a non-rigid surface registration method which computes the correspondence between two surfaces accurately and efficiently. The cover tree based hierarchical clustering and NN search were utilized to reduce the search space for correspondence points in ICP. This reduced the computational complexity of the correspondence computation. In addition, registration accuracy of the proposed method is better than the methods using conventional clustering, especially in the noisy dataset. The proposed negative Jacobian term of energy function led to registration with less deformation folding. Extending cover tree construction to consider orientation of the surface points introduced a hybrid similarity measure for ICP that allows capturing more reliable correspondence points.

A cover tree-based hierarchical clustering reduced the search space of the correspondence candidates of each point on S from all points on T to only $(1-c^{4d})/(1-c^4)$ of the points, where d is the depth of the sub-tree that corresponds to a cluster. Therefore, the complexity was reduced from $O(n^2)$ to $O(n(1-c^{4d})/(1-c^4))$. Proof of this reduction can be found in appendix 1. In addition, a cover tree-based NN search found the k correspondence candidates of every point on S from the points on T. The search space of the correspondence computation for a point was limited to k and the complexity was reduced to $O(c^{12}\ n \log n)$. The proposed cover tree based NN search was not compared with the other NN search algorithms such as k-d tree or v-p tree. In the future, we consider doing this comparison.

We proposed an optimization function for non-rigid ICP algorithm, including fitting term, stiffness term, and Jacobian term. The proposed optimization function with Jacobian penalty term regularized the deformation so that the resulted surface has smooth deformation with less folding. The results showed that the proposed method led to the smallest ratio of the negative Jacobian compared to the other non-rigid ICP methods. The results also showed that the ratio of the negative Jacobian was reduced by incorporating proposed negative Jacobian term.

One interesting result was that the proposed method showed the best results in CT-Kinect datasets in aspects of registration accuracy and percentage of the negative Jacobian. The Microsoft Kinect camera has relatively poor perception accuracy for the depth and thus the reconstructed surface from the depth map was very noisy and bumpy. This result demonstrated that the cover tree based hierarchical clustering was suitable for the noisy datasets. We improved the registration accuracy by taking into account the distribution and orientation of the point for tree construction.

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APPENDIX

Claim 1: The correspondence computation using cover tree-based hierarchical clustering reduces the time complexity of Cluster-based ICP from $O(n^2)$ to $O(n(1-c^{4d})/(1-c^4))$ where c is the expansion constant of the cover tree and d is the depth of the sub-tree that corresponds to a cluster.

Proof:

We know that each node in the cover tree has at most c^4 children (Beygelzimer et al. 2006). Assume l is the number of points in the largest cluster. Let's assume the worst case, when the constructed cover tree is balanced and each node has exactly c^4 children. Cutting the cover tree at level i with k nodes, each cluster contains one root node of the sub-tree and all its decedent nodes in all the lower levels from the level i down to the leaves level j. Let d denote the depth of the sub-tree, i.e. $d = j-i$. The number of the nodes in each cluster is calculated as follows: At level i, d is 0 and each cluster has one root node. The total number of nodes at level i is $(c^4)^0 = 1$. At the next level i-1, d is 1 and each cluster has at most c^4 nodes which are the children of the root node. The total number of nodes at level i-1 is $(c^4)^1 = c^4$. As the level decreases by 1, d increases by 1 and each cluster at each level has at most $(c^4)^d$ nodes. Therefore, the total number of the nodes in a cluster is calculated using Eq. (1).

$$\sum_{i=0}^{d} c^{4i} = (c^4)^0 + (c^4)^1 + (c^4)^2 + \ldots + (c^4)^d$$

$$= (1-c^{4d})/(1-c^4)$$

Thus, the number of the nodes l in the largest cluster is upper bounded by $(1-c^{4d})/(1-c^4)$ and the time complexity of ICP is upper bounded by $O(n(1-c^{4d})/(1-c^4))$ ■.

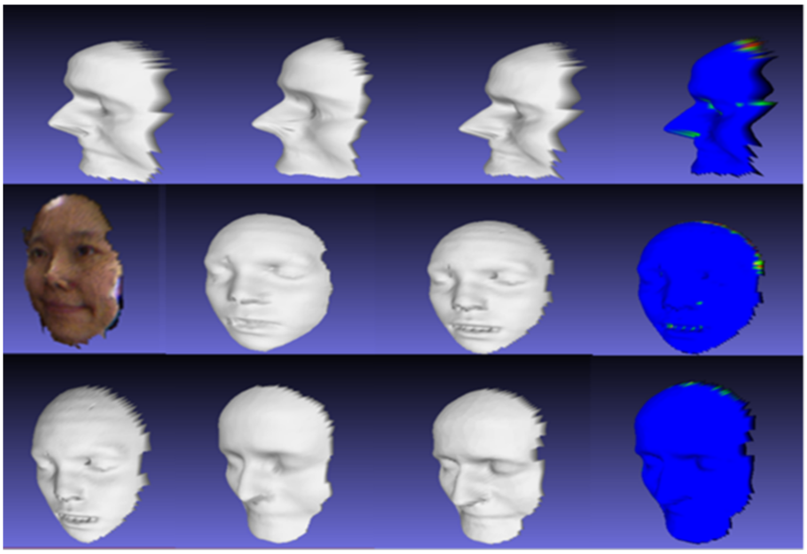

Figure 1: The registration results of the proposed method for simulated dataset (first row), Kinect-CT dataset (second row), and CT-CT dataset. The initial source surfaces S (leftmost) were registered to the target surfaces T (3rd column) by cluster-based ICP and non-rigid ICP. The differences between the deformed surface (2nd column) and the target were represented to color-coded error map (4th column).

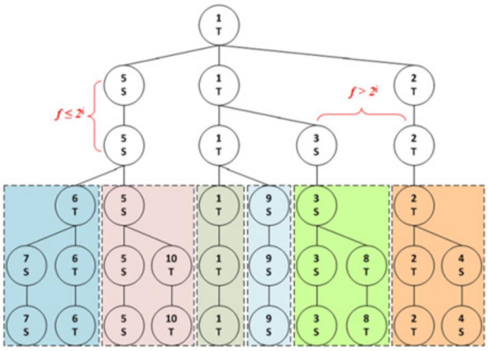

Figure 2: Example of clustering ten points using cover tree. When we cut the tree at depth equals to 3, the ten points are clustered into six clusters as indicated by shaded squares. The points of each cluster correspond to the leaf nodes of each sub-tree. The number and the character written in each node indicate the order of insertion and the original surface that this point belongs to, either T or S.

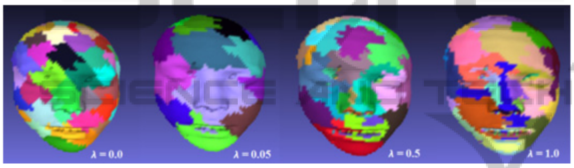

Figure 3: The effect of the λ weight of the angular term in the cover tree construction can be shown by clustering at depth equals to 3 in a face dataset. Left image: Traditional cover tree constructed with distance only. Three images in the right: Proposed cover tree construction with distance and angular term with different λ weights.

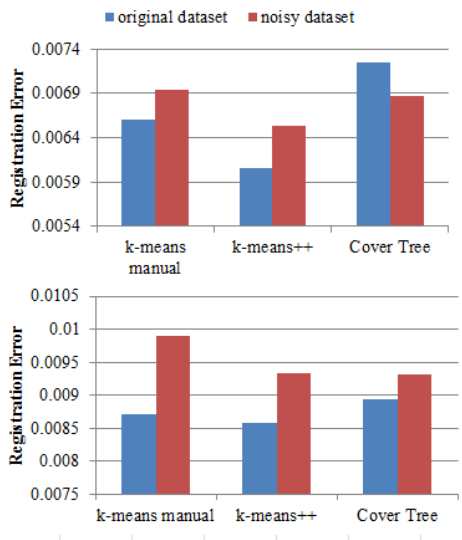

Figure 4: The comparison of the registration accuracy between original and noisy datasets: simulated dataset (top) and CT-CT dataset (bottom).

Table 1: The number of surface points of the three tested datasets.

	Number of points in S	Number of points in T
Simulated CT Dataset	3067	2906
CT-Kinect Dataset	4591	3145
CT-CT Dataset	3145	4076

Table 2: The percentage of the points with negative Jacobian when applying 4 different non-rigid ICP methods to three datasets. The proposed method was tested with and without applying Jacobian term by adjusting the weighting factor δ.

(unit: %)	Naive ICP	ICP with k-means manual	ICP with k-means++	ICP with cover tree (δ=0)	ICP with cover tree (δ=1)
Simulated CT Dataset	13.8	10.23	9.83	9.22	**8.21**
CT-Kinect Dataset	19.98	15.09	16.67	**10.97**	11.16
CT-CT Dataset	6.68	6.75	6.05	5.07	**4.61**

Table 3: Processing Time of Clustering and Correspondence Computation.

(unit: sec)		k-means manual	k-means++	Proposed clustering
Simulated CT Dataset	Clustering	2.153	1.857	**0.936**
	Correspondence (cluster-based ICP)	**0.501**	0.516	0.577
	Correspondence (non-rigid ICP)	**69.685**	212.645	81.370
CT-Kinect Dataset	Clustering	1.185	0.921	**0.671**
	Correspondence (cluster-based ICP)	0.967	0.531	**0.530**
	Correspondence (non-rigid ICP)	309.569	150.104	**107.547**
CT-CT Dataset	Clustering	1.560	2.714	**0.686**
	Correspondence (cluster-based ICP)	0.451	0.313	**0.280**
	Correspondence (non-rigid ICP)	122.040	**75.926**	78.405