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Characterization of polyhedron monotonicity

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Abstract

The notion of polygon monotonicity has been well researched to be used as an important property for various geometric problems. This notion can be more extended for categorizing the boundary shapes of polyhedrons, but it has not been explored enough yet. This paper characterizes three types of polyhedron monotonicity: strong-, weak-, *and directional-monotonicity*: (Toussaint, 1985). We reexamine the three types of polyhedron monotonicity by relating them with 3D manufacturing problems, and present their formulation with geometric problems on the sphere.

Keywords: 3D manufacturability; Polyhedron monotonicity; Strongly-monotone; Weakly-monotone; Directionally-monotone

1. Introduction

Orienting models or equipments in many manufacturing processes such as NC machining, mold casting, and layered manufacturing has been an essential geometric problem in the automation of manufacturing industry [1–6]. This paper investigates a nice geometric notion for finding feasible directions in manufacturing processes, which is the *polyhedron monotonicity* reflecting the boundary shapes of 3D designs. The notion of monotonicity is slightly different to that of visibility; they are equivalent in 2D when the viewer in visibility moves infinitely. The monotonicity can be used for classifying the boundary shapes in manufacturing since it is just the pure representation of themselves.

A polygon is said to be *monotone* in a direction d, if every line parallel to d intersects its boundary at most two times. The notion of polygon monotonicity has been well researched since it can be used for designing efficient algorithms for some geometric problems. Given a simple polygon, an $O(n)$ time algorithm for testing its monotonicity has been presented by Preparata and Supowit [7], where a polygon is said to be *simple* if adjacent edges intersect only at a common vertex and there is no pair of non-adjacent edges sharing a point. Sack and Toussaint: [8] proposed a linear time algorithm for computing all directions for disassembling a pair of simple polygons by using the polygon monotonicity; two polygons are movably separable with a single translation in directions d and $-d$, respectively, if both of them are monotone in the direction d.

Unlike the polygon monotonicity, we can not find many researches about polyhedron monotonicity. The polyhedron monotonicity can be defined diversely since the boundary shapes of 3D objects are more complicated than those of 2D objects. This paper discusses about the properties of polyhedron monotonicity by relating it with 3D manufacturability problems, and presents methods for determining the three types of polyhedron monotonicity: strong-, and weak-, and directional-monotonicity [9].

2. Overview of polyhedron monotonicity

2.1. Notion of monotonicity

First, we define the notion of monotonicity of polygons and surfaces. In this paper, a polygon (surface/chain) is said

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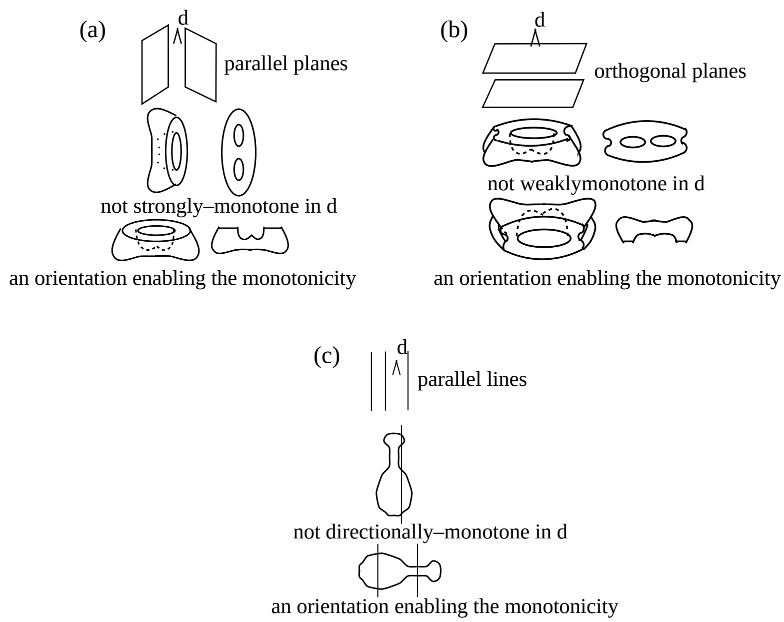

Fig. 1. Orientation of 3D objects versus the three types of monotonicity.

to be monotone in d, if every line parallel to d has at most two (one/one) intersections with the polygon (surface/chain). The geometry of polyhedrons is too complicated to directly extend this notion into them. Toussaint [9] has suggested that we can categorize the monotonicity of polyhedrons into three types as illustrated in Fig. 1, which are defined as follows:

- A polyhedron $\wp$ is *strongly-monotone* in a direction d, if the intersection of $\wp$ with every plane parallel to d is a monotone polygon in d.
- A polyhedron $\wp$ is *weakly-monotone* in a direction d, if the intersection of $\wp$ with every plane orthogonal to d is a simple polygon.
- A polyhedron $\wp$ is *directionally-monotone*[1] in a direction d, if every line parallel to d intersects the boundary of $\wp$ at most two times.

The three types of monotonicity are related with each other as presented in Fig. 2. A convex polyhedron has all types of monotonicity in every direction. If $\wp$ is strongly-monotone in a direction d, $\wp$ is also directionally-monotone in a direction d, and weakly-monotone in all directions orthogonal to d. However, the converse is not true. There is no relation between the weak- and the directional-monotonicity.

2.2. Relation to 3D manufacturability

The polyhedron monotonicity reflecting the boundary shape is closely related with 3D manufacturability. Suppose that we have designed the mechanical components of a new electric home appliance. The components are manufactured with various technologies such as NC machining, mould injection, layered manufacturing, and so on. Next they will be assembled together to form the entire structure of the electric home appliance.

The strong-monotonicity is related with the disassemblability problem encountered in the automatic generation of an assembly planning of 3D components. A sequence of assembly operations is the reverse of a disassembly sequence on the 3D components when only translation movements are allowed. Two polyhedrons that are strongly-monotone in the same direction can be disassembled by a single translation [9]. However, it has been an open question whether two polyhedrons that are strongly-monotone in directions different to each other can also be disassembled[2]. This paper gives a counter example for this open question; two polyhedrons can be locked even though they are strongly-monotone in directions θ and ϕ, respectively, as given in Fig. 3(b). Hence, the directions in which two components are strongly-monotone in common are partial solutions since they are *sufficient* for the disassemblability.

A weakly monotone direction can be used for orienting a model in gravity casting with just one pin gate [1]. The weak-monotonicity is also a *preferred* condition on layered manufacturing [10] that slices a model by a set of parallel planes and manufactures it layer by layer by tracing out the contours of slices as illustrated in Fig. 4(a). It is natural to ask which direction is better for the setup of a model. The shape of contours for each slice is an important criterion

[1] This term is used instead of 'directionally-convex' in [9], which is the simple extension of polygon monotonicity.

[2] Given two polygons $\wp_1$ and $\wp 2$ that are monotone in directions θ and ϕ, respectively, $\wp 1$ and $\wp 2$ are disassemblable in at least one of two directions $\theta+\pi/2$, $\phi+\pi/2$ [9].

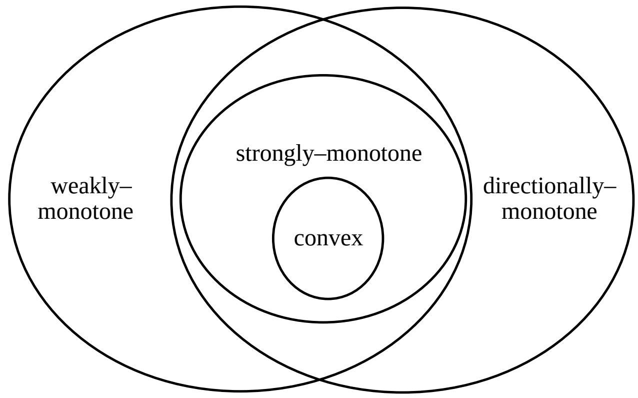

Fig. 2. Relations among three types of monotonicity.

for the setup direction since the manufacturing process is accomplished by scanning the contours or hatching the facets of the slice. Hence, the weakly-monotone directions of the model can be used for setting up the model since the intersection of the model with every plane orthogonal to the directions generates a simple contour.

The directional-monotonicity is a useful property for the mould design for a model. Since two plates are mostly used to form a seal in mould casting, the first step of automatic mould design is to investigate a feasible direction along which two plates forming a seal can be separated. However, two plates may not be removed due to some portions of moulded piece called undercuts that have to be dealt with devices such as side cores [1,4]. If a polyhedron is directionally-monotone as illustrated in Fig. 4(b), its surface is divided into two sub-surfaces that are separable in the monotone directions. Hence, the directional-monotonicity is a *necessary and sufficient* condition for the separability of two plates in mould design that needs no side core[3].

3. Characterization of polyhedron monotonicity

3.1. Geometry on the sphere

We define notation and review geometric primitives on the sphere. Let E^d be the d dimensional Euclidean space and $p=\{x_1,\ldots,x_d\}$ be a point in E^d. Then, the space on the boundary of the unit sphere centered at origin is described as

$$\mathcal{S}^{d-1} = \{p|||p|| = 1\}.$$

A point p on $\mathcal{S}^2$ is a unit vector in E^3. A *great circle* on $\mathcal{S}^2$, which is determined by the intersection of $\mathcal{S}^2$ with a plane containing the origin, is a set of points

$$GC(p) = \{x|px = 0, x \in \mathcal{S}^2\}.$$

[3] Chen et al. [4] have developed nice algorithms for minimizing the number of side cores in mould design, whose solutions are sufficient but not necessary.

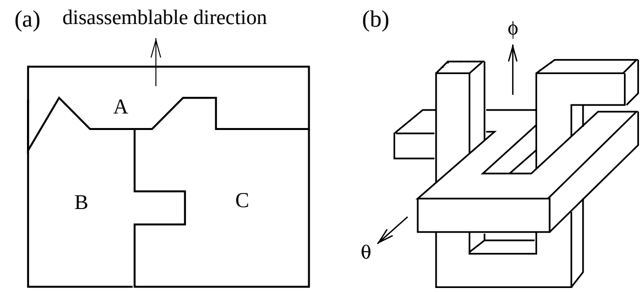

Fig. 3. Disassemblability.

A *hemisphere* that is the spherical region bounded by a great circle is a set of points

$$HS(p) = \{x|px \geq 0, x \in \mathcal{S}^2\}.$$

Lemma 1. $HS(p)$ contains $U=\{u_1,\ldots,u_n\}$, if and only if $p \in \cap_{i=1}^{n} HS(u_i)$: ([14]).

As illustrated in Fig. 5, let U be the set of outward unit normal vectors a surface S. Clearly, U is a set of points on $\mathcal{S}^2$, which is called the *Gaussian map* of S. The set of directions visible to S can be described with $\cap_{i=1}^{n} HS(u_i)$, which is known as the *visibility map* of S. The spherical convex hull of the Gaussian map, which is the boundary of the convex set of U, will be denoted by $GCH(U)$. On the while, the spherical convex hull of the visibility map $\cap_{i=1}^{n} HS(u_i)$ will be denoted by $VCH(U)$. For the brief expressions, S and $-S$ in $GCH()$ and $VCH()$ will be interpreted as U and $-U$, respectively, where $-U$ is the set of inward unit normals of S. For example, $GCH(S)$ and $VCH(S)$ mean $GCH(U)$ and $VCH(U)$, respectively.

Lemma 2. Two spherical convex hulls $GCH(U)$ and $VCH(U)$ are dual to each other; a vertex v_i of $GCH(U)$ for all i corresponds to an edge (*arc*) e_i in $VCH(U)$, and vice versa [5].

3.2. Strong-monotonicity

Let $\wp$ denote a simple polyhedron. We represent the geometric operations of convex hull and regularized difference: [11] with $CH()$ and $-^*$, respectively. The operation $-^*$ is similar to the set-theoretic difference, but it eliminates dangling low-dimensional structures such as line segment. The operation of $\wp -^* CH(\wp)$ will produce other polyhedrons $\mathcal{D}_i$ for $i=1,\ldots,k$, which are called the *deficiencies* of $\wp$. The surface of $\mathcal{D}_i$ is composed of two kinds of sub-surfaces: the part of $\wp$ and that of $CH(\wp)$, which are called the *pocket* and the *lid*, respectively. The pocket and the lid of $\mathcal{D}_i$ will be denoted by P_i and L_i, respectively. Note that all faces of a pocket are connected but those of a lid may not be connected. The examples of pockets and lids of polyhedrons are presented in Fig. 6.

The strong-monotonicity of $\wp$ is limited by the deficiencies of the polyhedron since a convex polyhedron

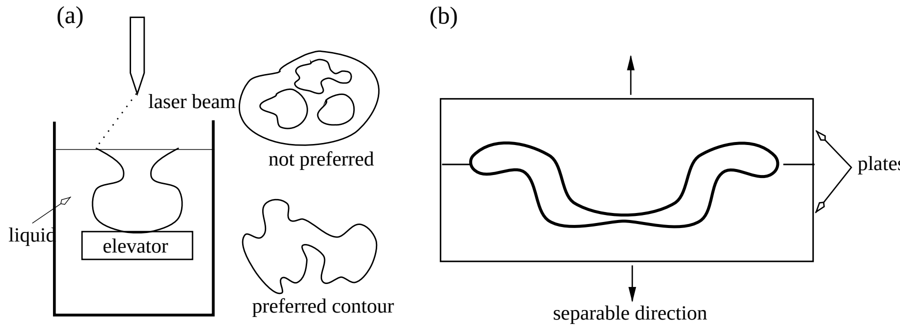

Fig. 4. Layered manufacturing and mould design.

is strongly-monotone in every direction. We characterize the strong-monotonicity of $\wp$ by exploiting the surface monotonicity of P_i and L_i for all $i=1,\ldots,k$.

Lemma 3. $\wp$ is strongly-monotone in a direction d, if and only if both of P_i and L_i for all $i=1,\ldots,k$ are monotone in d.

Proof. Let $PPL(d)$ denote an arbitrary plane parallel to the direction d. The symbol '$\wedge$' denotes the intersection of two objects in 3D space.

For proving the sufficiency part with the contradiction, we assume that P is not strongly-monotone in d when P_i and L_i for all $i=1\ldots k$ are monotone in d. If $\wp$ is not strongly-monotone in d, there is a plane $PPL(d)$ such that $\wp \wedge PPL(d)$ generates a non-monotone polygon in d, or two or more polygons. In the former case (Fig. 7(a)), let A be the non-monotone polygon. Clearly, we can get at least a non-monotone chain in d from $A-{}^*CH(A)$, which is the part of a pocket P_i for some i. Hence, there is a non-monotone P_i in d since a line parallel to d can intersect P_i two or more times. The latter is divided into two cases (Fig. 7(b) and (c)); a polygon contains another polygon or the polygons are disjointed. If a polygon contains another polygon, the internal polygon is the part of a pocket P_i for some i, i.e. P_i is not monotone in d. Otherwise, the line segments a and b supporting them are the part of a lid L_i for some i, i.e. L_i is not monotone in d. This is the contradiction.

The necessity part can be proved also with the three cases of Fig. 7. Assume that P_i or L_i for some i is not monotone in d when $\wp$ is strongly-monotone in d. If P_i is not monotone in d, there is a plane $PPL(d)$ such that L_i generates a non-monotone chain in d as illustrated in Fig. 7(a) and (b). On the other hand, if L_i is not monotone in d as illustrated in Fig. 7(c), there is a plane $PPL(d)$ such that $L_i \wedge PPL(d)$ generates two or more disjointed chains. This is because L_i is a convex surface if its faces are connected, and there is a hole otherwise. We can make such examples by intersecting some vertical planes with the cases of Fig. 6(c) and (d), respectively. This is the contradiction. □

The monotone directions of P_i and L_i can be characterized by the unit normals of faces forming them since a face with a unit normal u is visible in a direction $d \in HS(u)$. Hence, we can formulate the strong monotonicity of $\wp$ as the following geometric problem on $\mathcal{S}^2$.

Lemma 4. $\wp$ is strongly-monotone in a direction d, if and only if $GC(d)$ separates $GCH(P_i \cup L_i)$ for all $i=1\ldots k$.

Proof. In general, a surface S may not be monotone in a direction $d \in VCH(S) \cup VCH(-S)$. However, a pocket P_i of a polyhedron is always monotone in d, if and only if $d \in VCH(L_i) \cup VCH(-L_i)$. A lid L_i is also monotone in d, if and only if $d \in VCH(L_i) \cup VCH(-L_i)$.

The set of monotone directions of P_i and L_i is described as $(VCH(P_i) \cup VCH(-P_i)) \cap (VCH(L_i) \cup VCH(-L_i))$. This is equivalent to $VCH(P_i \cup L_i) \cup VCH(P_i \cup -L_i) \cup VCH(-P_i \cup L_i)$. $VCH(P_i \cup L_i) = VCH(-P_i \cup -L_i)=$, since they are, respectively, the set of outward unit normals and the set of inward unit normals of a deficiency D_i. Thus, the set of monotone directions of P_i and L_i for all $i=1\ldots k$ is represented as $\cap_{i=1}^{k}(VCH)(P_i \cup -L_i) \cup VCH(-P_i \cup L_i))$.

By Lemma 1 and 2, a direction d is included in $VCH(U)$ if and only if $HS(d)$ contains $GCH(U)$. Thus, for a direction $d \in \cap_{i=1}^{k}(VCH(P_i \cup -L_i) \cup VCH(-P_i \cup L_i))$, $HS(d)$ contains $GCH(P_i \cup -L_i)$ or $GCH(-P_i \cup L_i)$ for any i. In other words, P_i and L_i are monotone in d, if and only if the great circle $GC(d)$ separates $GCH(P_i \cup -L_i)$ for all $i=1\ldots k$. By this result and Lemma 3, the proof has been finished. □

The pockets and lids of a polyhedron can be obtained in $O(n \text{ lo } n)$ time, where n is the number of all faces of the polyhedron [11,12]. All great circles separating k spherical polygons can be found $O(mk \log k)$ time, where $m=O(n)$ is

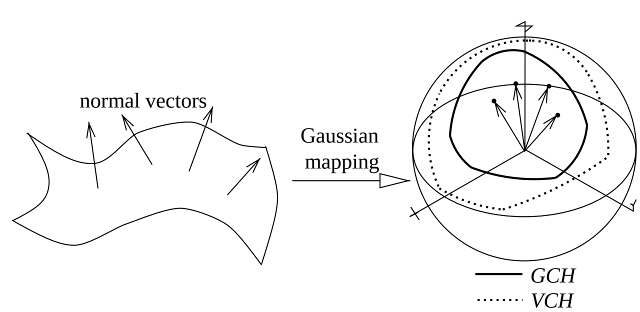

Fig. 5. Convex hulls of a Gaussian map and the corresponding visibility map.

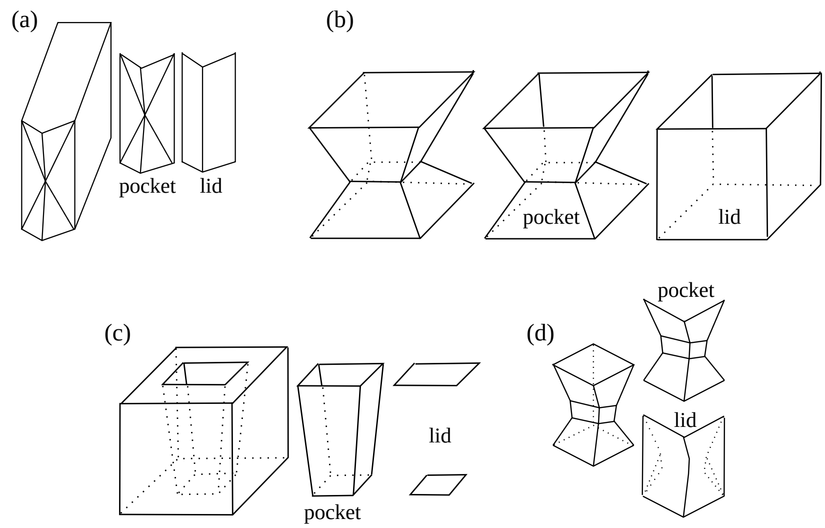

Fig. 6. Pockets and lids of polyhedrons.

the number of all vertices [3]. Thus, we get a result for the strong-monotonicity as.

Theorem 1. All directions for the strong monotonicity of a polyhedron can be computed in $O(nk \log k + n \log n)$ time, where n and k are the numbers of all faces and all deficiencies of the polyhedron, respectively.

3.3. Weak- and directional-monotonicity

The two types of the weak- and the directional-monotonicity of a polyhedron are limited also by its deficiencies. In order to characterize the two types of polyhedron monotonicity, we define other sub-surfaces from the deficiencies.

We have defined the pockets and the lids of $\wp$ by computing its deficiencies from the operations of $\wp -^* CH(\wp)$. A set of faces forming a pocket can be partitioned into several sets of faces by sets of convex edges that are connected, which will be called *sub-pockets* and denoted by SP_i for all $i = 1,\ldots,k_s$. An edge is said to be *convex* if the exterior dihedral angle between its two incident faces is greater than 180°. Now we define another sub-surface called a *sub-lid* (but this is not a pure sub-part of a lid). From the operation of $SP_i -^* CH(SP_i)$, we get a set of faces that are not contained in $\wp$, which is the sub-lid corresponding to SP_i and will be denoted by SL_i. The examples of SP_i and SL_i are illustrated in Fig. 8.

We assume that $\wp$ has no hole in the formulation of the weak-monotonicity, since a polyhedron with a hole is not weakly-monotone in any direction. Then, we can describe an obvious characterization for the weak-monotonicity of $\wp$ with SP_i as.

Lemma 5. $\wp$ is weakly-monotone in a direction d if and only if SP_i for all $i = 1,\ldots,k_s$ is divided into at most two sub-surfaces by every plane orthogonal to the direction d.

Let SLF_{ij} for all $j = 1,\ldots,l_i$ be each face $\in SL_i$, each of which will also represent its unit normal in $HS()$ similarly to $GCH()$ and $VCH()$. Then, we can formulate a geometric problem on $\mathcal{S}^2$ for the weak-monotonicity of $\wp$ as.

Lemma 6. $\wp$ is weakly-monotone in a direction d, if and only if $GC(d)$ intersects $VCH(SP_i \cup -SLF_{ij})$ for all $i = 1,\ldots,k_s$, $j = 1,\ldots,l_i$.

Proof. Let $OPL(d)$ denote an arbitrary plane orthogonal to the direction d. Note that $\{OPL(d)\} \subset \{PPL(d')\}$ if d' is orthogonal to d, where $\{OPL(d)\}$ is the set of all planes orthogonal to d and $\{PPL(d')\}$ is the set of all planes parallel to d'.

We consider the condition that a given SP_i for some i is divided into more than two sub-surfaces by a plane. First, SP_i is divided into more than two sub-surfaces by a plane

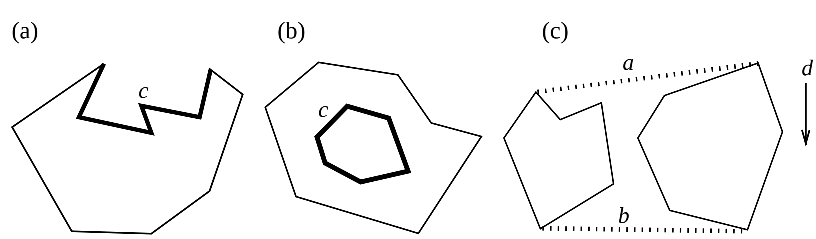

Fig. 7. The proof of Lemma 3.

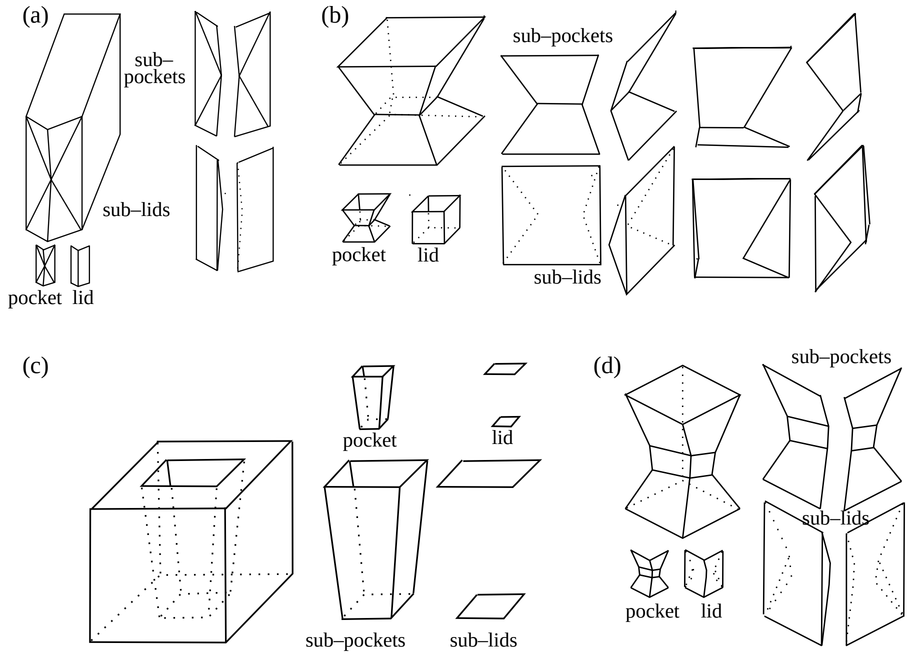

Fig. 8. Sub-pockets and sub-lids of polyhedrons.

$\in \{PPL(d')\}$ if $d' \notin VCH(SP_i)$, as illustrated in Fig. 9(a). Second, even when $d' \in VCH(SP_i)$, SP_i is divided into more than two sub-surfaces by a plane $\in \{PPL(d')\}$ if $d' \in HS(SLF_{ij})$ for any $j=1,\ldots,l_i$, as illustrated in Fig. 9(b).

Hence, the set of planes that does not divide SP_i into more than two sub-surfaces is described as $\bigcap_{j=1}^{i_s}\{PPL(d'_{ij})\}$, where $d'_{ij} \in VCH(SP_i \cup -SLF_{ij})$ that is equivalent to $d'_{ij} \in VCH(SP_i)$ and $d'_{ij} \notin HS(SLF_{ij})$. Since the directions orthogonal to d are on $GC(d)$ and $\{PPL(d'_1)\} \cap \{PPL(d'_2)\} = \{OPL(d)\}$ for $d = d'_1 d'_2$ (d is orthogonal to both of d'_1 and d'_2), $\{OPL(d)\}$ does not divide any sub-pockets into more than two pockets if and only if every $d'_{ij} \in VCH(SP_i \cup -SLF_{ij})$ is on $GC(d)$. In other words, $\wp$ is weakly-monotone if and only if $GC(d)$ intersects $VCH(SP_i \cup -SLF_{ij})$ for all $i=1,\ldots,k_s$, $j=1,\ldots,l_i$. □

The directional-monotonicity of $\wp$ can be characterized by exploiting the surface monotonicity of SP_i for all $i=1,\ldots,k_s$ as.

Lemma 7. $\wp$ is directionally-monotone in a direction d, if and only if SP_i for all $i=1,\ldots,k_s$ are monotone in d.

Proof. For proving the sufficiency part with the contradiction, assume that $\wp$ is not directionally-monotone in d, when SP_i for all $i=1,\ldots,k_s$ are monotone in d. Then, there is a line parallel to d such that it intersects a pocket of $\wp$ two or more times. Let a and b, respectively, be the two points that are the first and the last intersection points of the pocket with the line. There are two cases; a and b are included in SP_i for some i, or not. In the first case, clearly, the SP_i including a and b is not monotone in d. In the second case, let SP_a and SP_b be the sub-pockets including a and b, respectively. The path from a to b on the pocket always passes a convex edge, since SP_a and SP_b are divided by a set of convex edges. Hence, there are at least two intersection points with a line parallel to d in each of SP_a and SP_b. It follows that neither of SP_a and SP_b is monotone in d, i.e. SP_i is not monotone in d.

For the necessity part, assume that SP_i for some i is not monotone in d, when $\wp$ is directionally-monotone in d. Then, SP_i has two or more intersection points with a line parallel to d, i.e. the line intersects $\wp$ more than two times. This is the contradiction. □

We can describe another geometric formulation on $\mathcal{S}^2$ for the directional-monotonicity of $\wp$ with the proof similar to that of Lemma 4 as.

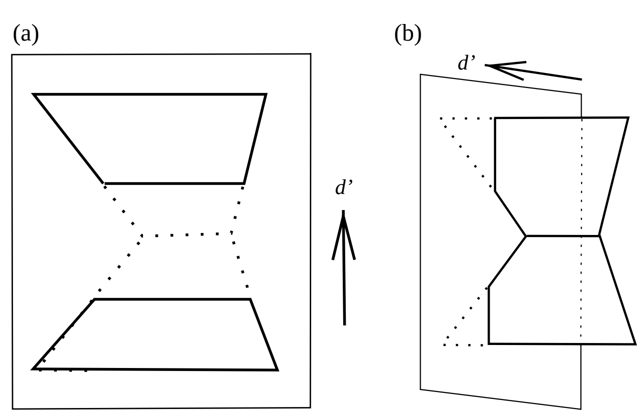

Fig. 9. The proof of Lemma 6.

Lemma 8. $\wp$ is directionally-monotone in a direction d, if and only if $GC(d)$ separates $GCH(SP_i)$ for all $i=1,\ldots,k_s$.

Prior to checking the weak-monotonicity, it can be determined in $O(n \log n)$ time to determine whether or not $\wp$ has any hole. We can divide a pocket into sub-pockets by searching a set of faces that are adjacent to each other and share a concave edge, which can be performed in $O(n)$ time similarly to the depth-first search or the breadth-first search of a graph [13]. Furthermore, all great circles intersecting k spherical polygons can be found also in $O(nk \log k)$ time [3]. Hence, we summarize the results for the weak- and the directional-monotonicity as.

Theorem 2. All directions for the weak-(directional-) monotonicity of a polyhedron can be obtained in $O(nk \log k+n \log n)$ time, where n and k are the numbers of all faces and all sub-pockets of the polyhedron, respectively.

4. Conclusion

We discussed about the properties of the three types of polyhedron monotonicity: strong-, weak-, and directional-monotonicity. The strong-monotonicity was reexamined with the disassemblability problem. Other types of polyhedron monotonicity are also related with 3D manufacturability problems such as layered manufacturing and mould casting.

By characterizing the monotonicity of a given polyhedron with the sub-surfaces of its deficiencies, the problem of determining the polyhedron monotonicity was formulated as a well-known geometric problem on the sphere; find great circles separating or intersecting a set of spherical polygons. Consequently, all directions for the strong- (weak-, directional-) monotonicity of a polyhedron can be obtained in $O(nk \log k+n \log n)$ time, where n and k are the numbers of all faces and all deficiencies (sub-pockets) of the polyhedron, respectively.

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